

Numerical Investigation of Wire Icing Mechanism Using Particle Method

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Abstract Wire icing by frozen rain in winter is harmful which would heavily increase the load of wires and even break down the power line and tower. Mechanism study of wire icing using numerical method is helpful to understand the phenomena and find out ways to reduce the hazard. A typical particle-based method, an improved moving particle semi-implicit (MPS) method was proposed to reproduce the process of heat transfer with phase change. The improved MPS method includes a heat transfer model to calculate the liquid/solid temperature field, and a phase transformation model to track the phase interface. The cases of droplet's impacting on a cold cylindrical wire, and the accumulation and solidification of multi-droplets on rigid surface were calculated and investigated. The results are consistent with the numerical and experimental data in previous researches. The validated models and methods were employed to study the mechanism of wire icing, important factors such as the temperature distribution, the solidify position and the surface wettability etc. were discussed.

1 INTRODUCTION

Freezing of a supercooled water droplet is a common natural phenomenon, which couples with heat transfer, phase change, and phase interface movement. In the field of power, freezing of supercooled water droplets on transmission lines poses a serious threat not only to power transmission but also to life safety. Severe ice accretion heavily increases the load of wires and even breaks down the power line and tower, which may result in property damage and casualties [1]. Various studies on freezing of droplets have been conducted to deepen the fundamental understanding and to predict or prevent from ice accretion in anti-icing applications.

Based on experimental observations on freezing of supercooled droplets, the freezing process can be divided into five distinct stages [2,3,4]: (1)liquid cooled(supercooled), (2)nucleation, (3)recalescence, (4)freezing, and (5)solid cooling. Nucleation is the start point of recalescence and phase change, at which the temperature of water is below equilibrium freezing point of 0 °C. Recalescence usually lasts tens of milliseconds, during which the temperature of the droplet rises to 0 °C because of a latent heat releasing, then the droplet becomes a uniform and opaque mixture of ice and water. At the freezing stage, freezing front driven by heat conduction advances from the preexisting nuclei [5] to the top of the droplet. Several experimental studies on the sessile water droplet on a horizontal cold plate have revealed the details of freezing mechanism. X. Zhang et al [6] statistically made the relationship between nucleation point and droplet size by a numerous of experiments. Nucleation point of supercooled droplet has randomness, satisfies the normal distribution, and scatters in wider range with smaller drop size. S. Suzuki et al and Y. S. Djikaev et al [7,8] pointed out that nucleation occurs preferentially on the solid-liquid-gas interface. F.

Tavakoli [9] drew a conclusion that recalescence speed depends heavily on the supercooled temperature. Tip singularity formation of sessile water droplet after freezing stage has been studied theoretically and experimentally by X. Zhang et al [10]. Based on the models for simulating the shapes of the solidified droplets separately established by Sanz et al [11] and Anderson et al [12], X. Zhang et al. took into account the effect of both the physical properties and the gravity effect on the droplet shape, built a theoretical model to simulate the droplet profile evolution. Dynamic and freezing mechanism of water droplets on a cold solid surface, which is of significant importance for anti-icing, also attracted the attentions. The impact and freezing processes of droplets on cold horizontal, inclined and spherical surface have been investigated [13,14,15]. In addition, vast of numerical investigations on liquid-solid phase change have been conducted. Using the front tracking technique combined with an interpolation technique, V.N. Duy et al [16] and T.V. Vu et al [17] separately simulated the freezing process of a water droplet attached on a vertical and horizontal cold plate under different Bond number to show the effect of it on water drop shape. W.W. Schultz et al used boundary integral method to study solidification of water droplet [18]. A.Virozub et al. mainly investigated the singular shape of frozen droplet by a classical Galerkin finite element method [19]. G. Chaudhary et al. used the enthalpy-based heat conduction equations to simulate the the freezing of static water droplets lying on supercooled surfaces and validated the numerical solution by comparison with their experimental result [20]. Based on OpenFOAM platform, Y. N. Yao et al. built a three-dimensional model including an extended phase change method to simulate the impact, spreading, and freezing of a water droplet on a cold surface [21], to deepen the understanding of the dynamic mechanism of this process. VOF(volume of fluid) method has been widely used to numerically investigate isothermal impact and solidification of a molten metal droplet on cooled plate [22-24].

In the field of power, wire icing is a complex process generally accompanied with impact, shearing motion, momentum dissipation and accretion of supercooled droplets on a cylinder surface. A Lagrange numerical method is an appropriate choice to reproduce the wire icing process and predict the ice morphology under various conditions. As one of Lagrange numerical method, SPH (Smooth Particle Hydrodynamic) has been used by M.Y. Zhang et al. to simulate spreading, splashing and solidification of yttria stabilized zirconia (YSZ) droplet after inclined impact [25]. Another Lagrange numerical method, MPS (Moving Particle Semi-implicit) method also has been widely used to simulate the heat transfer natural convection [26] and solidification of molten metal in nuclear engineering [27-30], but not too much researches of supercooled water droplet freezing used MPS method can be found in literatures. In this study, the MPS method with the phase change model was employed to simulate the freezing process of a steady droplet on a cold plate. The comparison between the experiment result and numerical result validates the correction of this algorithm. Then, the process of multi-droplet falling on the cold cylinder was simulated with different supercooled temperature and wettability.

2 MOVING PARTICLE SEMI-IMPLICIT (MPS) METHOD

2.1 Governing equations

Governing equations of MPS the method are continuity, momentum and energy

conservations as follows:

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} + \mathbf{f}_s \quad (2)$$

$$\frac{Dh}{Dt} = k \nabla^2 T + Q \quad (3)$$

where \mathbf{u} is velocity, t is time, ρ is density, p is pressure, μ is the kinetic viscosity coefficient, \mathbf{g} is the acceleration of gravity, \mathbf{f}_s is surface tension translated into a force per unit fluid volume, h is enthalpy, k is thermal conductivity, T is absolute temperature, ∇ is the gradient, and ∇^2 is the Laplacian.

2.2 Discretization

In the MPS method [31], numerous interacting particles are employed to reproduce the continuous flow field and simulate the fluid motion based on discretized governing equations. The interaction between two particles is calculated using kernel function, the particle number density n_i of a fluid particle i is defined to remain a constant n_0 during solving process to ensure the incompressibility of fluid. Gradient operator and Laplacian operator in the governing equation are discretized in the MPS method. Use these models, the gradient and Laplacian of physical quantity φ on particle i can be obtained as the summation of quantity φ on particles in its effect radius with a weight function w .

More details of the MPS method are introduced in the literature by S. Koshizuka et al. who first proposed the MPS method [31].

2.3 Surface tension model

According to the definition that surface tension can be considered as the macro performance of an intermolecular force, Kondo et al [33] introduced an inter-particle potential model as a source term of governing equations to simulate the surface tension of fluid.

The following equation describes the inter-particle potential model:

$$P_{\text{tension}}(r) = \begin{cases} \frac{1}{3} C \left(r - \frac{3}{2} l_{\min} + \frac{1}{2} l_{\max} \right) (r - l_{\max})^2 & (r < l_{\max}) \\ 0 & (r \geq l_{\max}) \end{cases} \quad (4)$$

where l_{\max} and l_{\min} are function parameters. l_{\max} and l_{\min} are $3.1l_0$ and $1.5l_0$, respectively [34]. As a correction parameter, C is related to the physical property of the liquid [32].

The partial derivative of P_{tension} with respect to r is the interparticle force \mathbf{f}_s :

$$\mathbf{f}_s = \frac{\partial P_{\text{tension}}}{\partial \mathbf{r}} \mathbf{n} \quad (5)$$

where \mathbf{r} is the distance between fluid particles, and \mathbf{n} is the normal vector.

2.4 Phase change model

The discretized energy conservation equation can be used to calculate the enthalpy of fluid particles, further to obtain their temperature based the function between temperature and enthalpy for different phases as follows:

$$\mathbf{T} = \begin{cases} \mathbf{T}_s + \frac{h - h_{s0}}{\rho C_{ps}} & (h < h_{s0}) \\ \mathbf{T}_s & (h_{s0} < h < h_{sl}) \\ \mathbf{T}_s + \frac{h - h_{sl}}{\rho C_{pl}} & (h_{sl} < h) \end{cases} \quad (6)$$

where \mathbf{T}_s is the equilibrium freezing point which is 0 °C, C_{ps} and C_{pl} are specific heat of solid and liquid state, respectively. h_{s0} and h_{sl} are the enthalpy of fluid at the start and the end point of freezing stage. In the algorithm of this study, a fluid particle whose temperature is less than \mathbf{T}_s is regarded as solid particle otherwise is seen as liquid particle.

3 ALGORITHM VALIDATION

3.1 One-dimensional unsteady heat conduction

One-dimensional unsteady heat conduction of semi-infinite plate is calculated by the MPS method in this study to validate the algorithm's correctness. Fig.1 (a) shows the model of the numerical example, the left side of this model is the heat source with the first type of boundary condition, its temperature T_b is constantly 300 K. The plate's initial temperature T_i is 100 K, as time progresses, temperature transfers along x direction in this plate. To meet the one dimension requirement, the width and height of the plate are separately 0.2 m and 2.0 m, the large aspect ratio 10 ensures the case as a quasi-one-dimensional issue. One-dimensional unsteady heat conduction's analytical solution shows as follow:

$$\frac{T(x, t) - T_b}{T_i - T_b} = \text{erf} \left(\frac{x}{2\sqrt{\alpha t}} \right) \quad (7)$$

where α denotes $k/\rho C$, k is heat conductivity, ρ is density and C is specific heat. In this case, α is 1.716×10^{-4} . Fig. 1 (b) presents the temperature distribution in x direction between numerical result and analytical result at 0.5 s, 1 s, 3 s, 5 s, 7 s. The comparison represents a good agreement between simulation and analytical result.

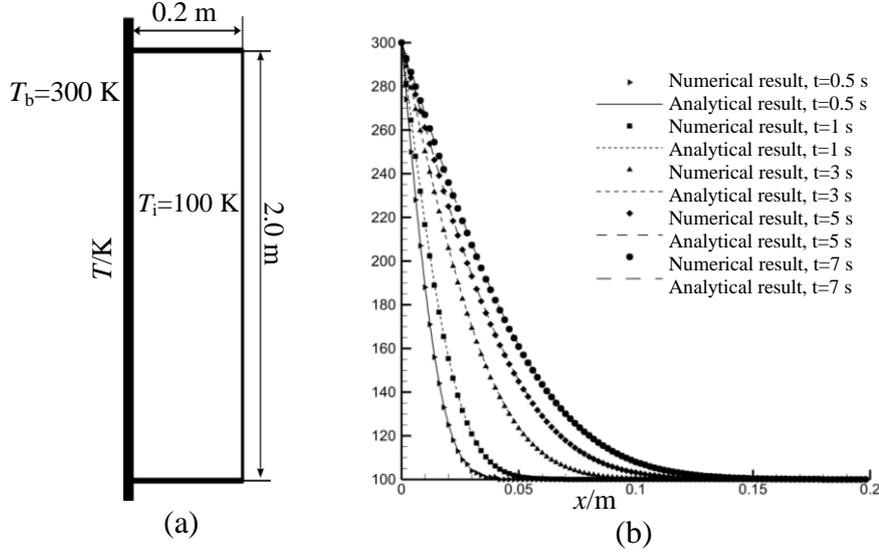


Fig.1 Model of one-dimensional unsteady heat conduction (a), temperature distribution along x direction of numerical solutions (symbols) and analytical solutions (lines) at 0.5 s, 1 s, 3 s, 5 s, 7 s (b).

3.2 Freezing of sessile droplet on cold plate

To verify the phase change model in this MPS algorithm, a 2D simulation of sessile water droplet freezing on cold plate is carried out. According to the experimental study of X. Zhang [6], a 10 μL water droplet on a horizontal plate whose temperature is $-16.5\text{ }^\circ\text{C}$ is conducted in this section to show the droplet profile and the solidification front of the freezing process. In the time series of the experimental result as show in Fig. 2, water droplet is stable and sessile on the plate. Phase change and solidification front advance start at 0.0 s, at this time, recalescence has finished and the temperature of the whole droplet rises back to $0\text{ }^\circ\text{C}$ due to some release of the latent heat. Thus, in this numerical case, the initial temperature condition is $0\text{ }^\circ\text{C}$, the physical property parameters of the materials are referred to the study of Zhang. X [6]. In addition, the heat transfer between the droplet surface and the atmosphere is ignored in the simulation.

Fig. 3 separately presents the comparison of the droplet's height and the distance from solidification front to plate between the experimental result and the numerical result.

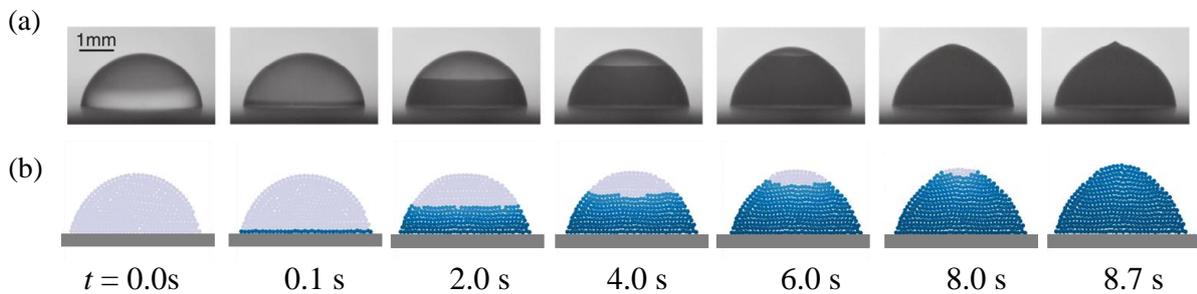


Fig.2 Comparison between the simulation in this study and the experimental result in the previous study [6] for freezing of a sessile droplet on a cold plate

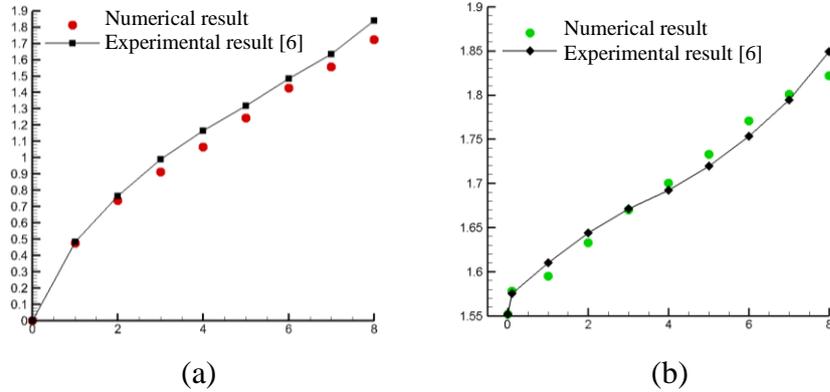


Fig.3 Comparison between the experimental result and the numerical result for the height of droplet (a) and the distance from solidification front to plate (b).

4 RESULTS AND DISCUSSION

Wire icing is a complex process coupled with movement and solidification of supercooled droplets. In the freezing rain, numerous supercooled liquid droplets adhere to on the surface of power facilities and change to solid phase, which lead to extensive ice layer accumulation as time progresses. In this section, to reveal the effects of supercooled temperature and wettability of cold surface on freezing rate, freezing shape and freezing area, the process of fifteen droplets falling on a cold surface and solidifying is simulated under the above conditions. Further, the mechanism of wire icing process can be understood more deeply from the solution of this simulation.

4.1 Effect of supercooled temperature

Two-dimensional droplets falling on a cylinder surface under supercooled temperature is considered. The supercooled temperature condition of the cylinder surface in this simulation is $-10\text{ }^{\circ}\text{C}$, $-20\text{ }^{\circ}\text{C}$, $-30\text{ }^{\circ}\text{C}$, separately. We assumed the end of recalescence as the initial status of the liquid droplets, because the nucleation stage and the recalescence stage are too much faster than freezing stage (in 10 to 20 milliseconds generally). Thus, the initial temperature of the liquid droplets is set to the fusion temperature of water, $0\text{ }^{\circ}\text{C}$. In the recalescence stage, with some of liquid turning to ice, the latent heat of supercooled liquid releases to raise the droplet's temperature to fusion temperature, then the droplet becomes uniform ice-water mixture, then solidifies as time progresses in the freezing stage. In this stage, there is a distinct solidification front evolving from cold boundary to the solidification termination position. The simulation is shows in Fig. 4 and Fig. 5.

Fig. 4 shows of first droplet falling on the cold cylinder surface, separately. The cylinder's radius is 11 mm, and droplet's radius is 1 mm, initial velocity is 0.1m/s in the vertical direction and initial falling height is approximately 5 mm to the top of the cylinder. In addition, the wall boundary condition of the cylinder surface is no-slip and no-penetration, its static contact angle is 75° , which corresponds to hydrophilic surface.

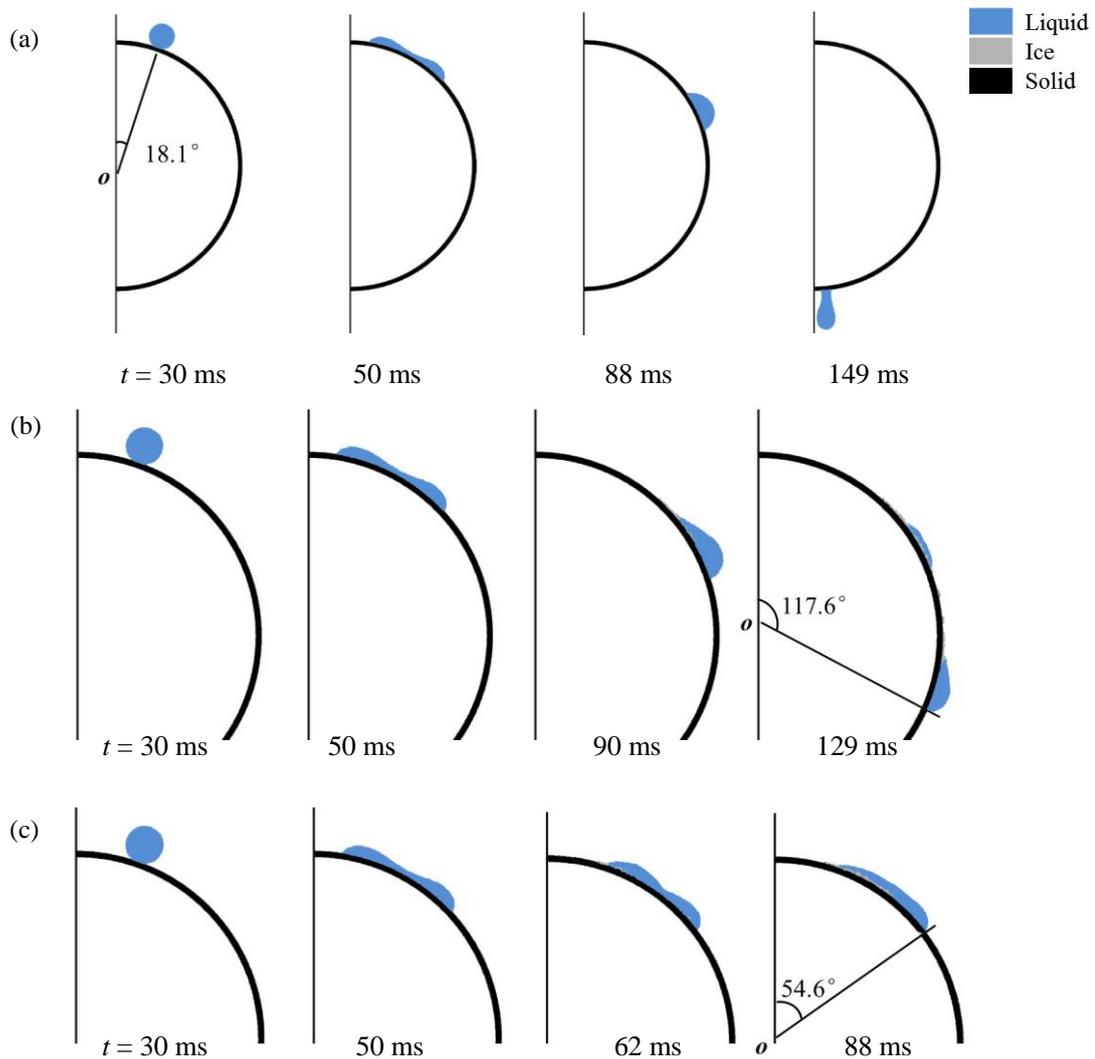


Fig.4 The process of the first supercooled droplet attaching and moving on the surface of the cylinder under - 10°C (a), -20°C (b) and -30°C (c).

As shown in Fig.4, from 30 ms to 50 ms, the droplet touches and wets the cylinder surface, the wetted area is the largest at 50 ms, and the droplet shows a hump shape due to the inertia force and surface tension, during this process, no ice particle occurs. Then, the droplet moves downward along the surface with retraction. Under -10°C temperature, the droplet always remains liquid phase before it moves to the bottom surface of the cylinder and separates from the surface. Under -20°C temperature, freezing stage starts at 90 ms, there is a layer of solidified particles on the solid surface, which reduce the velocity of the whole droplet. When t is 129 ms, the liquid stops flowing, the solidification front will evolve from the layer of solidified particles to the top of the liquid. Under -30°C temperature, a similar freezing stage starts about 30 ms earlier than it is under -20°C temperature, before the liquid retracts to spherical shape.

To estimate the ice covered area under -20°C and -30°C , the angle from leading edge of the droplet A_e is measured, separately. As shown in Fig.4 (b), A_e is 117.6° under -20°C , in

Fig.4 (c), A_c is 54.6° under -30°C .

Fig.5 presents the final morphology of multi-droplets solidification on the surface of cold cylinder under -10°C , -20°C and -30°C , separately. There are fifteen droplets alternately falling on the point A and point B of the cylinder surface. With a low supercooled temperature (-10°C), during flowing along the cold wall, only a few of the droplet solidifies and adheres to the wall. With increasing supercooled temperature, the enthalpy of a liquid drops faster to reach h_{sl} . At -20°C , liquid stops flowing at 1.35 s and a stable liquid film forms on the surface of the cold wall, there are two hanging drops hang at the leading edge of the liquid film. Then, the steady liquid film is completely frozen to ice at 13.70 s. Under -30°C temperature, liquid stops flowing at 1.30 s, then at 4.50 s, the steady liquid film is completely frozen to ice.

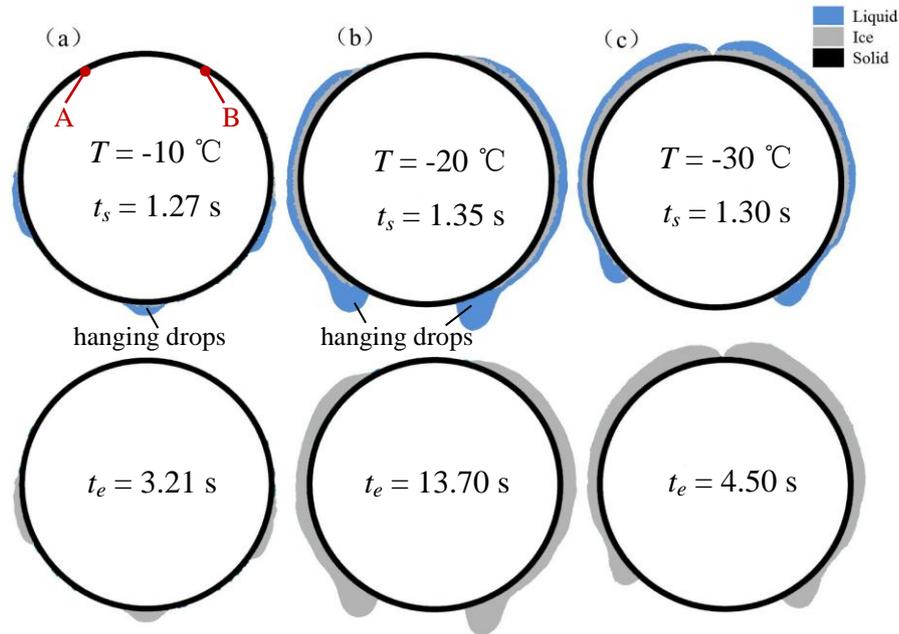


Fig.5 The state at which flow stops (the first row), and the state at which solidification stops (the second row) under -10°C (a), -20°C (b) and -30°C (c).

4.2 Effect of wettability of wire surface

The wettability of the solid surface also plays a significant role in freezing stage. Fig. 6 presents the feature of a single supercooled droplet moving and solidifying on the cold cylinder surface with different contact angles. In Fig.6 (a), the contact angle of the solid surface is 45° corresponding to hydrophilicity. Droplet on this surface has a large wettability area, which prompts the heat transfer between liquid and cold surface. The freezing stage starts at 100 ms with the initial solidified particles occurring. The momentum dissipation caused by an increasing number of solidified particles reduces the velocity of liquid, at 129ms, the whole liquid stops flowing, A_c under this condition is 54.6° . In Fig.6 (b), contact angle is set to 90° , which is the critical value of hydrophilicity and hydrophobicity. It can be seen that the deformation of the droplet caused by impinging on the solid wall surface quickly recovers. Little solidified particles occur during the droplet moving along the right

semicircle surface, the droplet separates from the solid surface at 165.6° . In fig.6 (c), contact angle is set to 135° as a hydrophobic surface, the droplet separates at 94.2° , and there is no solidified particle under such condition.

Therefore, a hydrophobic surface has been used widely in de-icing industry.

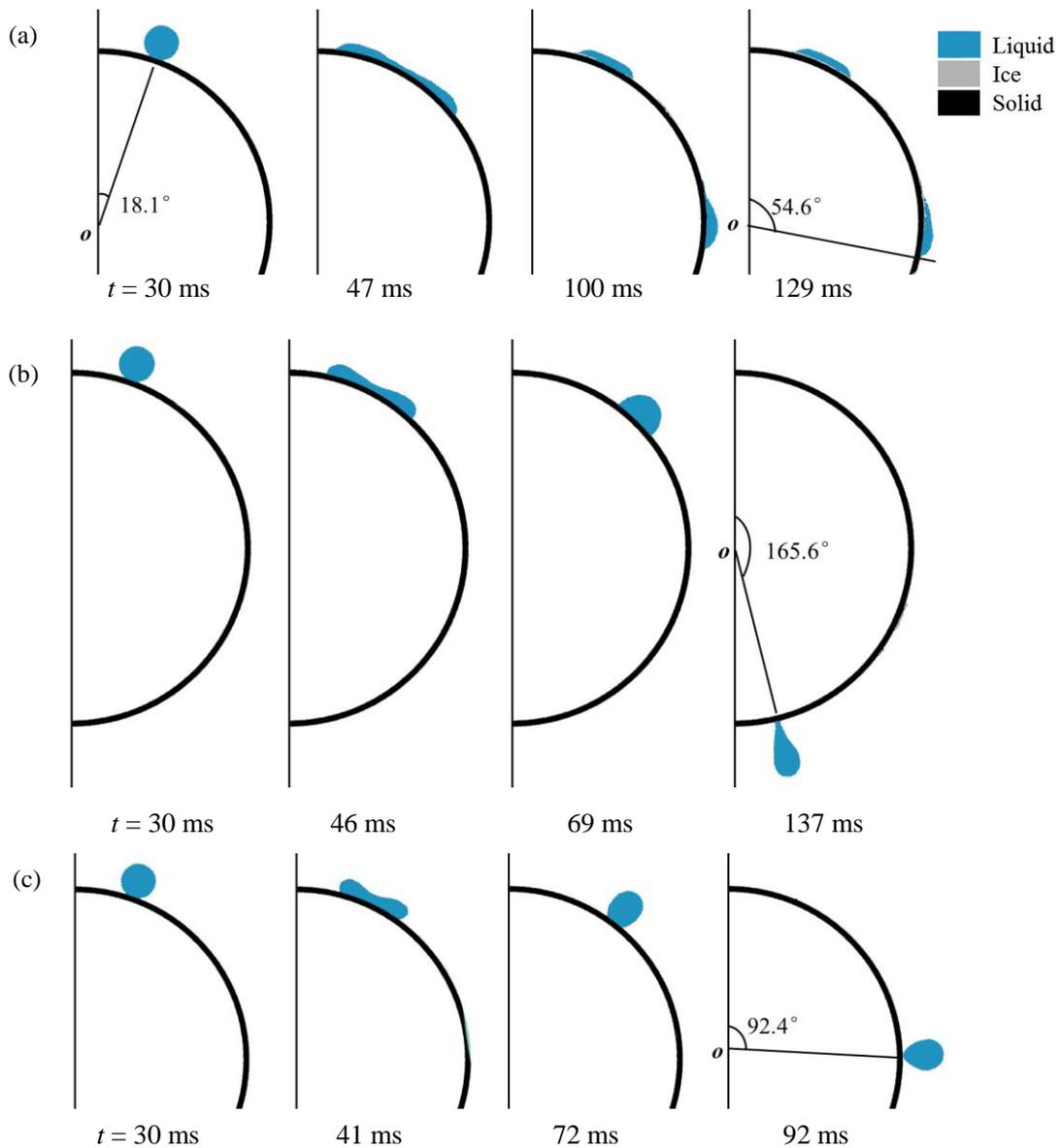


Fig.6 The process of the first supercooled droplet attaching and moving on the surface of the cylinder with different contact angle, 45° (a), 90° (b) and 135° (c).

Fig.7 presents the final morphology of multi-droplets solidification on the surface of cold cylinder under different contact angle CA, 45° (a), 90° (b) and 135° (c). Higher wettability of liquid leads to the larger contact time, contact area, and faster heat transfer. Thus, when CA= 45° , liquid film can form on the surface of the wall, then is frozen to an ice layer after 11.12 seconds. When CA= 90° , only a few of liquid over comes the gravity and adheres to the wall, which even cannot form a continuous film. When CA= 135° , which represents a

hydrophobic surface, the attraction between the droplets and the solid wall is very small, each droplet separates from the wall quickly before it solidifies.

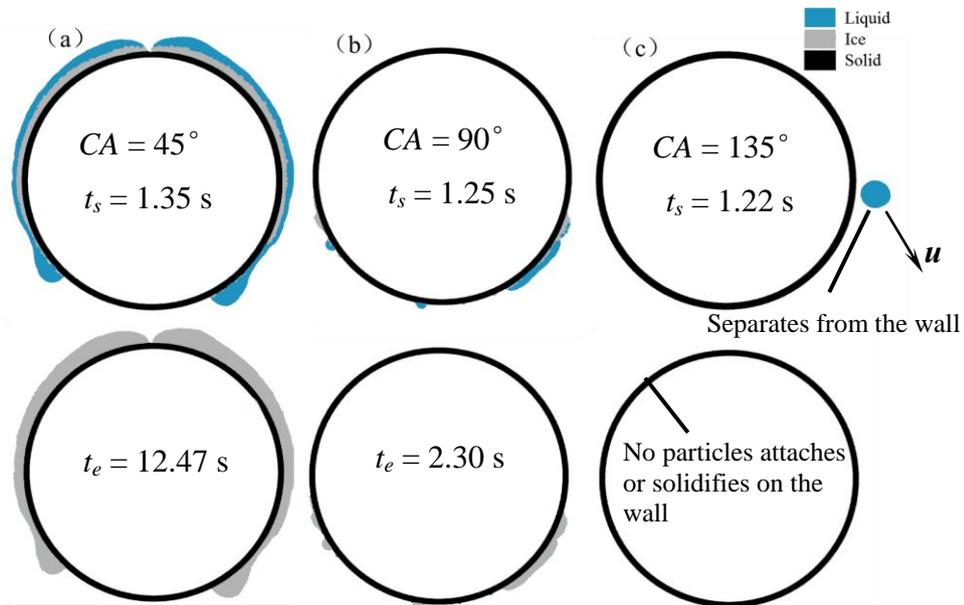


Fig.7 The state at which flow stops (the first row), and the state at which solidification stops (the second row) when CA=45° (a), 90° (b) and 135° (c).

5 CONCLUSION

A two-dimensional numerical investigation of supercooled droplet solidifying on a cold wall by moving particle semi-implicit method with heat transfer model and liquid-solid phase change model has been presented in this study. The conical point due to the volume expansion can be simulated by this algorithm which has been validated by comprising the droplet morphology with the experiment result. We considered the multi-droplets falling on a cold cylinder wall under different temperature and wetting conditions. To reveal the effect on freezing characteristics, we varied the temperature (-10°C, -20°C, -30°C) and contact angle (45°, 90°, 135°) of cylinder surface. With the decreasing temperature, liquid particles are frozen faster on the cold solid surface. With increasing contact angle of solid wall, the contact time and contact area between liquid and solid is smaller, which leads to slower heat transfer efficiency, so a steady liquid film can form on the cold wall when the CA is 45°, while only a few of liquid can finally adhere to the cold wall when the CA is 90°, and when the CA is 135°, the droplets quickly disengage from the wall due to the inertia force, no particles attaches to the cold wall in the end.

This study shows a viability to investigate the liquid-solid phase change problem coupled with flowing, provides a certain basis for understanding the mechanism of wire icing problem.

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