NUMERICAL TECHNIQUES FOR THE FAST GENERATION OF SAMPLES USING THE PARTICLE REFINEMENT METHOD

Yaseen U Sharif[1], Matteo O Ciantia[1], Michael J Brown[1], Jonathan A Knappett[1] and Jonathan Ball[2]

[1] University of Dundee
Fulton Building Dundee DD1 4HN UK
yusharif@dundee.ac.uk

[2] Roger Bullivants Ltd
Burton Upon Trent Derbyshire DE11 9DU UK

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Abstract. With the recent technological advancements in terms of both hardware and software, DEM has increased in popularity among geotechnical engineers. This is due to its advantages when modelling large deformation problems over other numerical tools. Procedures have been created to enable fast generation of samples, but these tools have yet to reduce the run times of the civil engineering boundary value problems they are created for. In this paper a methodology has been described in which the particle refinement method and the periodic cell replication methods have been combined to create samples that are not only fast to generate but also reduce the run time of common geotechnical simulations, without reducing the resolution of the results obtained. An example sample is generated in this study and validated by modelling two geotechnical centrifuge pile installation tests.

1 INTRODUCTION

The discrete element method (DEM) can be used to model large deformation geotechnical problems, due to its ability to model the behaviour of granular materials such as rockfill [1] ballast [2] and sands [3]. By recording the movement and contact forces experienced by particles, insights into the behaviour of granular soil bodies can be investigated and explained. As a result, DEM is able to provide an insight into the complex behaviour for a variety of different boundary value problems (BVP) including cone penetration tests [4], standard penetration test [5] slope stability[6] and pile plugging[7]. When this is combined with the recorded data and observations made during physical modelling tests (1g, centrifuge and field scale), a greater understanding of the mechanistic behaviour of the geotechnical problem can be achieved.

A common problem that is investigated, in both DEM and physical modelling, is the penetration of a rigid object into body of soil, such as a cone penetration test or the installation of a pile. These problems require very large sample sizes to avoid boundary effects[8], increasing the number of particles in a sample to many million. This in turn leads to very long sample generation times and run times of the BVP. A common way around this is to increase the scaling of the particle size distribution (PSD) of the soil by a constant number (N). This reduces the number of particles in the sample by n^3 and therefore, the run time of both the sample generation and the BVP. To achieve manageable numbers of particles the scaling value is usually large, which can cause a loss in resolution of the resulting BVP as the ratio between the diameter of the pile (B) and the median particle size (d_{50}) reduces causing large voids to appear at the interface between the particles and pile. It is therefore ideal that the scaling value applied to the particles is as low as possible, but as previously discussed the smaller the scaling value the longer the computational time. To assess the computational efficiency of simulations the computational time will be used as an assessment. All simulations were performed using the same hardware (intel i7-6700 with 16GB of RAM) and software PFC3D 5.0.39 [9]

2 METHODOLOGIES FOR DECREASING COMPUTATIONAL TIME

To combat this problem of long computational times for sample generation Ciantia *et al*[10] presented the periodic cell replication method (PCRM), Figure 1. PCRM uses relatively small representative element volumes called cells, as a building block for construction of a large sample. The cells are consolidated to the required parameters of the final sample, i.e. a homogeneous voids ratio, with a stress state that reflects the base of the final sample. The cell is then replicated and stacked to the required height and the contact forces resolved. The stress state within the sample can then be scaled according to requirements and be used to testing BVPs. This method drastically reduces the time to create a sample, as consolidating a small slice is much easier than a large sample but the run time of the BVP is not reduced as this is predominately based upon the total number of the particles in the final sample.

McDowell *et al*[11] proposed the particle refinement method (PRM) similar to that of mesh refinement conducted in finite element analysis (FEA) (Figure 2). PRM uses small particle scaling in the central core of a sample with an increased scaling at the boundary of the sample. This allows for the particles that are in contact with the pile to be sufficiently small to provide a high resolution for results, while also decreasing computational time through a reduction of total particles in sample. To reduce the possibility of particle migration from core of low scaling to high scaling at the perimeter, buffer zones are implemented. The buffer zones consist of particles that have scaled to an intermediate amount, so that the voids within this zone are not large to allow merging of the various particle scaling. As all of the particles have the same properties, the bulk behaviour of the sample remains the same. To test whether the response of the soil body using PRM was as expected McDowell et al[11] conducted CPT within samples using a single particle size and one using four zones of particles of different sizes. The response of the cone resistance in the samples show a good comparison, with no migration between layers. The method showed large reductions in computational times for modelling a CPT conducted in a calibration chamber, but little guidance on size of buffer layers and increases in particle scale in these layers is given.

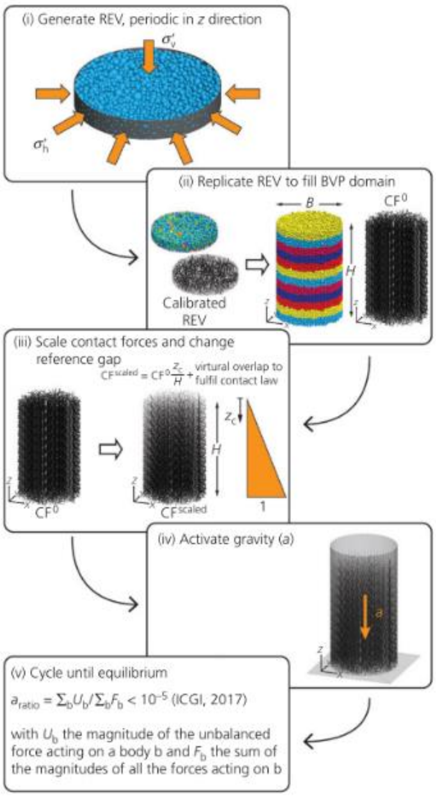


Figure 1: Flow chart outlining the steps to initialize a sample using the periodic cell replication method [10]

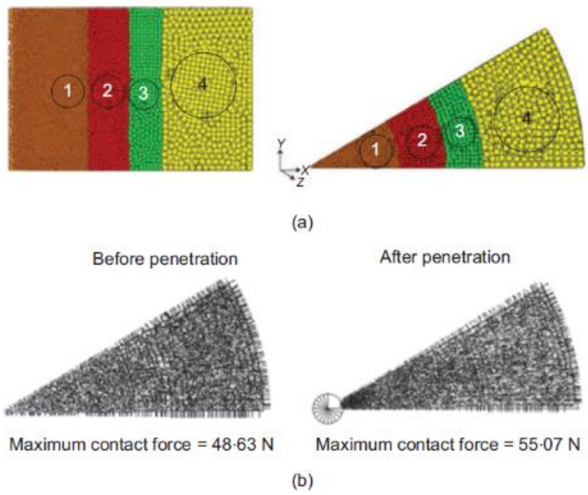


Figure 2: Example use of the particle refinement method a) particle scale locations. b) contact forces within the sample[11]

By combining these two sample generation methodologies it is possible to reduce not only the computational time required to create the sample but also to run the BVP with a high resolution. As no guidance on layer sizes and scaling is given, a methodology for this has been developed using the particle size distribution (PSD) of a sand as the basis for calculating the geometric properties of the soil sample. An example of the methodology is given below, using HST95 sand which is a medium to fine well graded quartzite sand that is commonly used at the University of Dundee. The behaviour and properties of the sand have been well documented for use with physical and numerical modelling. The properties of the particles and for the rigid walls are the same as those used in Sharif *et al* [12] and can be seen in Table 1. To assess whether the sample is able to replicate the results of physical tests, a comparison between the axially jacked and rotary jacked installation of a pile, installed into dense sand, in a series of centrifuge tests[13] was conducted.

Table 1: HST95 physical and DEM properties [12]

HST95 silica sand property	Value
Physical properties	
Sand unit weight γ (kN/m^3)	16.75
Minimum dry density γ_{max} (kN/m^3)	14.59
Maximum dry density γ_{min} (kN/m^3)	17.58
Critical state friction angle, φ (degrees)	32
Interface friction angle, δ (degrees)	18
D_{30} (mm)	0.12
D_{60} (mm)	0.14
DEM Parameters	
Shear modulus, G (GPa)	9
Friction coefficient, μ (-)	0.264
Poisson's ratio, ν (-)	0.2
Interface friction coefficient [pile], μ_{pile} (-)	0.16

3 METHODOLOGY

The initial step for creating the sample is to define the scaling required for the various regions of the soil body. To do this an initial core radius is required, as well as the radius for the complete sample and the scaling for the central core. The radius of the central core should be at least 2.5 times greater than the radius of the pile. This is to allow for a sufficient zone for the penetrating object and the transfer of force from the pile through the various scaled regions. The outer radius of the sample is highly dependent on the type of simulation that is to be conducted and usually is based upon the values used within physical and other numerical testing techniques. For this example, the central core diameter was chosen to be 12.5mm and the outer radius was chosen as being 250mm based upon size of the strong box used within the centrifuge tests. This value is larger than the minimum of 10B recommended by Bolton *et al* [8]. The radius of the pile to be used is 5mm.

The scaling of the PSD for the central core, should be calculated based upon the required B/d_{50}. A value of greater than 3 is recommended by Arroyo *et al* [14] . As the pile diameter is 10mm and the d_{50} of HST95 is 0.14mm a scaling of 10 times the standard PSD was chosen for the central core of the sample to give a B/d_{50} of 7.04. As PSD is being used rather than a single particle radius, a percentage increase in scale is required to reduce the possibility of particle migration. The percentage increase was determined based upon the d_{50} divided by the d_{min}. This would mean that the d_{min} of the larger scale would be smaller than the median particle size of the smaller scale, and thus avoid merging between zones. For the PSD of HST95 d_{min} = 0.09mm and as stated previously d_{50} = 0.14mm. This gives an increase in particle scaling of 1.56 times. This was then selected as 1.5 for reasons of simplification. The scaling (n) of each region (x) can then be calculated using equation 1.

$$n_{(x+1)} = n_x * {d_{50}}/{d_{min}} \quad (1)$$

The width of each of the buffer zones is now required. The zones must be wide enough to allow for each zone to model the bulk properties of the soil. The value of this was then chosen based on the layer height used in the PCRM, in which the layer height is usually around 2.2 times the largest particle diameter (d_{100}) in a given layer. This value was increased to $3n_{max}$ d_{100} to reduce the possibility of arching forming in the radial direction between layers.

$$t_x = 3n_x d_{100} \tag{2}$$

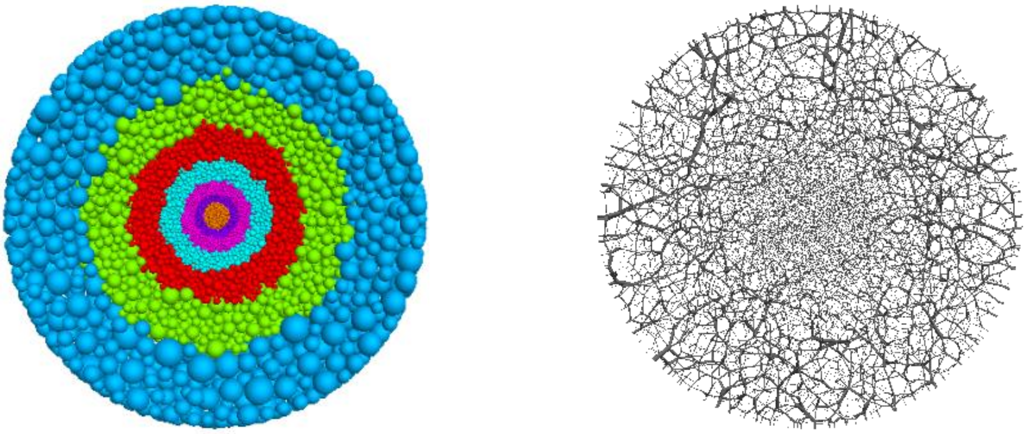
Where t is the thickness of a layer, n is the scaling in the layer, and x is the layer which is to be considered. The height of the periodic cell was kept as 2.2 times the largest particle diameter in the sample. This is 2.2 times the d_{100} of the largest scaling that is used in the sample.

Table 2: Dimensions of cell for use PRM

Zone (x)	**1**	**2**	**3**	**4**	**5**	**6**	**7**
PSD scale (N)	10.0	15.0	22.5	33.75	50.63	75.94	113.9
N*d_{100} (mm)	2.13	3.20	4.79	7.19	10.78	16.17	24.26
N*d_{50} (mm)	1.41	2.12	3.17	4.76	7.14	10.71	16.06
N*d_{00} (mm)	0.90	1.35	2.03	3.04	4.56	6.83	10.25
Layer thickness (mm)	15.60	10.86	16.29	24.44	36.66	54.99	82.49

With all the dimension of the sample now known, Table 2, the initial cell can be created. To create the particles, the radius expansion method (REM)[15] was used (Figure 3a). The particles were created, to a voids ratio (e) looser than required, in groups based upon their given scale, with the largest scale being created first and the smallest last. Once the particles were generated using REM, the contact forces in the sample were scaled to a small number, ensuring that the virtual overlap between the particles had been adjusted accordingly. Scaling of the contact forces at an early stage results in a sample that can be isotropically compressed to low a stress without having to dissipate excess forces generated by the very large particles during the REM process.

With the stress state within the cell now at an acceptable level for consolidation, the methodology for creating the sample follows that of the PCRM. The cell is isotropically consolidated to a mean normal stress (p'_0) of 10 kPa to a target voids ratio (e_0). For the case of this example a dense sample was required to match the sand used in the centrifuge tests, so the initial voids ratio was targeted to be 0.56 to give a relative density of 70%. The target initial voids ratio is then monitored through measurement spheres located within each scaled region and a global voids ratio was calculated considering the whole cell. To reduce the possibility of arching forming within the contact forces of the particles, the frictional value was cycled between 0 and 0.264. This allows the particles to slide past one another if large normal forces, indicative of arching, are generated.



a b

Figure 3: Example of a cell produced with varying scales. a) A layered soil using the dimensions from Table 2, b) Contact forces of the cell once isotropic consolidation has occurred

When the cell had reached the target conditions, the stress state on the cell was increased to match the final conditions of the bottom layer of soil for the soil chamber that is required. To model the installation of the pile a sample 0.4m tall was required at 50g. This results in a required vertical stress of 332kPa and a radial stress of 162 kPa. This increase in stress does not change the voids ratio of the sample (as it is already dense) but increases the confining stresses on the boundaries of the sample. The cell is then replicated to the desired height and removed from the periodic domain.

If a calibration chamber is required a rigid boundary is placed at the top and bottom of the cylinder and particle – wall contact forces are scaled according to the desired stress. For samples that replicate field, centrifuge or 1g testing the contact forces within the sample are scaled according to the gravitational level and the particle's vertical position within the sample. Thus, giving a gradient of stress within the sample.

To create the sample that is used as an example in this paper, 30 hours (real time) was required from start to finish. The total number of particles within the sample was 250,000. This time is relatively low in comparison with a sample that would be created using a single PSD scaling of 10. A sample with the same dimension using a single PSD scaling of 10 was not created, as the time required to do this and run the BVP is extremely large. The number of particles required to generate the sample would be in the 10,000,000 range.

4 TESTING

To assess whether the methodology described is able to replicate and be used to model large deformation BVP, the sample was used to model two geotechnical centrifuge tests[13]. The first test was a monotonic axially jacked straight shafted pile and the second was a rotary jacked installed pile of the same dimensions. To model the centrifuge tests using DEM a 10mm diameter pile with a cone apex angle of 60^{o} was created out of rigid boundaries. A simplified Hertz-Mindilin contact model was used to model the skin friction of the pile with the parameters the same as used in Sharif *et al* [12]. The pile was then installed into the soil to a depth of 200mm and the results for the installation torque and force compared with those of the centrifuge tests[13]. The rate of installation was kept low to reduce the possibility of the simulation becoming dynamic rather than quasi static. A vertical velocity of 0.2m/s was chosen as this is well below the limit of 0.8m/s shown by Tran et al[16].

From the comparison seen in Figure 4, it can be seen that using the results of the DEM simulations match the physical tests in the dense soil sample, for both the monotonic axially jacked pile and the rotary jacked pile for both the vertical force and torque during installation. From this it can be surmised that the soil body using both the PCRM and PRM methods is able to create a homogenous soil sample with the same bulk properties as the soil being modelled.

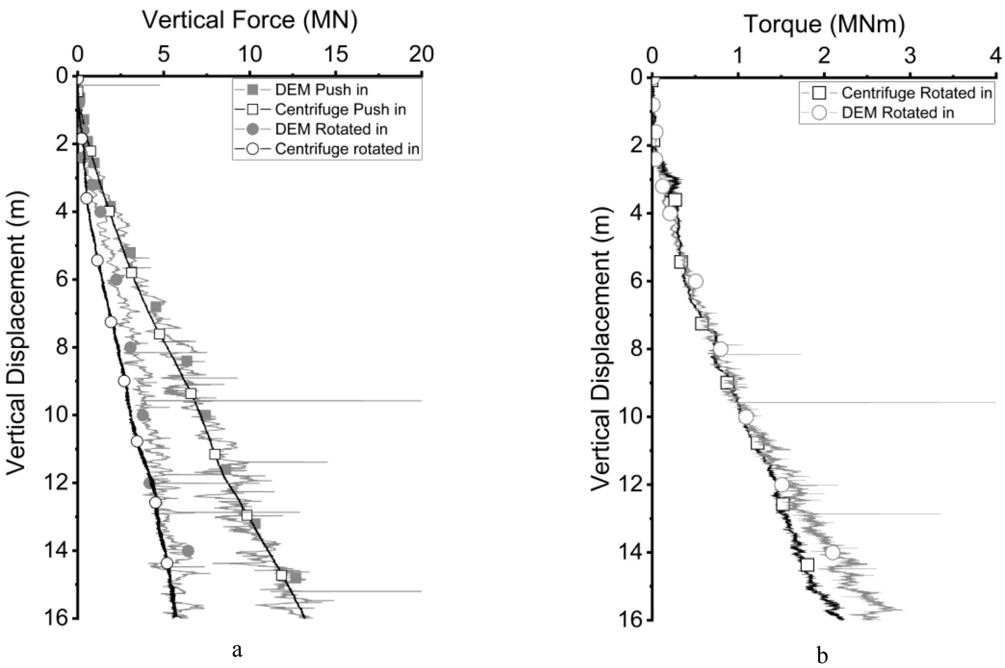


Figure 4: Geotechnical Centrifuge tests vs DEM for the installation of axially jacked and rotary jacked piles. a) Vertical force vs vertical displacement, b) Torque vs vertical displacement [12]

To model the installation of the pile 23 hours of computation time was required. This relatively low run time makes DEM simulations ideal for use as a tool for parametric testing of BVP. For example, the time taken to run a series of tests using small modifications in pile geometry would be must faster using DEM than if the same tests were to be conducted in a geotechnical centrifuge. The DEM simulations would also have the added benefit of being able to use the exact same soil for each test, reducing the possibility of small variations occurring in real sample preparation.

5 CONCLUSION

In this paper an original approach for fast initialisation and running of BVP using DEM has been introduced. The approach builds upon PCRM by introducing the use of PRM to reduce the computational time of large-scale geotechnical problems. Through the use of cell REVs with varying PSD scaling, consolidated in the periodic space, large scale soil samples can be created with that give high resolution results with a low computational cost. By basing the parameters for the geometric properties of the sample on the PSD of the sand to be used, the methodology can be applied to any common sand used in laboratory tests.

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