MICROMECHANICAL STUDY ON THE COMPRESSION BEHAVIOUR OF SAND-RUBBER PARTICLE MIXTURES CONSIDERING GRAIN-SCALE DEFORMABILITY

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Abstract This paper presents simulations of systematic confined compression tests on sand-rubber particle mixtures with varying rubber content using the DEM. The sand grains are modelled by using the clump logic in order to accurately represent their shape. The rubber particles are modelled using a cohesive bond model which allows to take into account grain-scale deformability, i.e., the spheres making up the rubber particles can move relative to each other. The simulations are calibrated and validated by systematically carried out experimental tests. Results of sand-rubber particle mixtures with different rubber contents are presented. The micromechanical investigations include coordination number, force chains and contact orientation histogram. The numerical predictions indicate that the coordination number increases significantly with increasing rubber content since the rubber particulate deform leading to larger number of contacts. The force chain and the contact orientation histogram are presented for the sand-sand contacts, sand-rubber contacts and rubber-rubber contacts separately to find the contribution of each contact type to the loading. The results show that for a rubber content of less than 20% the samples exhibit sandy behaviour and for rubber contents larger than 60% the behaviour is very similar to the one of pure rubber particle samples.

1 INTRODUCTION

The increasing growth in vehicle usage over the last decades led to a huge accumulation of scrap tires. Various options of recycling and reusing the scrap tires have been investigated. Very often they are used as construction material where whole tires are used for retaining walls, breakwater materials, crash barriers and sound barriers. In some cases the whole tires are cut into smaller rubber particles, so-called tire derived additives (TDA), in the form of shreds, chips, crumb or powder and then mixed with soil. Understanding the mechanical properties of such mixtures is crucial for an efficient geotechnical design [1].

Sand-rubber particle mixtures generally have favourable engineering properties such as high permeability, high endurance, appropriate strength, appropriate stiffness, and high
damping characteristics. These properties have mainly been investigated by experimental testing. Only recently, numerical models have been used to study the mechanical behaviour of sand-rubber particle mixtures. The heterogeneous nature of such mixtures, the big difference between the mechanical properties of the two materials and the very specific shapes of the rubber particles lead to a behaviour that is hard to predict by continuum methods such as the Finite Element Method (FEM). Nevertheless, the Discrete Element Method (DEM) is able to capture all these characteristics. The DEM is a numerical method that uses a granular approach to simulate materials and subsequently can easily consider large deformations and material heterogeneity [2]. In the last decade, DEM has also been applied to investigate the behaviour of sand-rubber particle mixtures [3,4,5]. The sand grains and rubber particle were either represented by discs in 2D or spheres in 3D. The deformability was taken into account by varying the contact stiffness. Only recently shape and grain-scale deformability have been taken into account properly [6,7]. Most of the studies focus on the macromechanical behaviour and more work is needed to better understand the micromechanics of sand-rubber particle mixtures.

In this paper, several one-dimensional compression tests on sand-rubber particle mixtures have been simulated using DEM in order to study its micromechanical behaviour. The unique deformability behaviour of the rubber particles is considered by modelling each rubber particle with an agglomerate of bonded spheres. This allows not only to take into account deformability but also shape. The sand grains are modelled by rigid agglomerates of spheres, i.e., clumps. This allows a more realistic representation of the real mechanical behaviour. A contact law verified by Asadi et al. [7] has been employed to model the interactions between the two materials. Section 2 summarises the main concepts of the numerical model. Section 3 discusses the results of the micromechanical study including evolution of coordination number, force chains and contact network histograms. Conclusions are provided in Section 4.

2 METHODODOLOGY

2.1 Numerical framework

The DEM uses Newton’s law to represent particle movements [2]. In the classical DEM, as implemented in the open-source framework YADE [8], which is used in this research, particles are represented by spheres. The spheres can be combined by rigid or deformable bonds to form more complex shapes. An agglomerate of rigidly bonded spheres is called clump. The interactions between the particles are described by contact laws which relate the local deformation (i.e., the particle overlap) to the contact force.

The contact law needs to be selected based on the geometrical and mechanical properties of the sample. One of objectives of this research is to consider the deformability of the rubber particles. Hence, a linear elastic cohesive contact model is applied between spheres forming the same rubber particle. The spheres forming the rubber particle can move relatively to each other when loaded and after unloading they come back to their original position. Tensile and compressive forces can be applied and the cohesive bond is considered unbreakable. The corresponding rheological model and force displacement diagrams are shown in Figure 1a. A non-cohesive linear-elastic plastic contact law is applied for all other contacts. This includes contacts between rubber particles, sand particles, and rubber and sand
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particles. The corresponding rheological model and force displacement diagrams are shown in Figure 1b. The detailed equations of the contact laws used in this study are reported in Asadi et al. [6]. It should be noted that it has been shown that the high difference in contact stiffness between the sand and rubber particles needs to be taken into account properly [7]. This is achieved by calculating the contact Young’s modulus $E_c$ based on Hertz’s theory:

$$E_c = \frac{E_1E_2}{E_1(1 - \nu_2^2) + E_2(1 - \nu_1^2)}$$

(2)

where $E_1$ and $E_2$ are the contact Young’s moduli of sand and rubber respectively, and $\nu_1$ and $\nu_2$ are the contact Poisson’s ratios of sand and rubber respectively. Note that the contact Poisson’s ratio is defined as the ratio of shear to normal contact stiffness.

Figure 1: Rheological models and corresponding force-displacement diagrams for: (a) cohesive contact for spheres forming the deformable rubber grain and (b) non-cohesive contacts between grains of the mixture.

2.2 Sample generation

Figure 2a shows a typical sand-rubber particle mixture used in the experimental tests [7]. The sand grains have an average diameter of 4 mm and are fairly round. The rubber particles are regular cubes with 4 mm side length. They have been prepared manually by cutting cubes from a 4 mm thick rubber band. The aim was to carry out systematic tests under controlled conditions. Hence, it was important to control size and shape of all particles. Figure 2b shows the numerical representation of the sand and rubber particles. The sand is
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represented by clumps made out of rigid agglomerates of overlapping spheres as can be seen by the yellow particles in Figure 2b. Three representative shapes were chosen and 4 to 7 different-sized spheres are used to approximate the real sand particle shape. The regular rubber particles are represented by a deformable agglomerate of 8 spheres of the same diameter (black particles in Figure 2b). It should be noted that the mean particle size in the numerical model was increased to 7 mm in order to keep computational costs at an acceptable level.

![Figure 2: Grains used in this study (after [7]): (a) typical sand-rubber particle mixture and (b) representative grains with corresponding representation as clump (sand) or cohesive aggregate (rubber).](image)

One-dimensional compression test simulations have been carried out for samples with different rubber content (RC). RC is defined as the volumetric rubber fraction of the mixture. Samples with RC values of 0 (pure sand mixture), 20, 40, 60 and 100% (pure rubber particle mixture) have been prepared. The oedometric cell with a diameter of 100 mm is represented by fixed triangular facets. The loading is applied by a wall moving at a constant loading rate of 3.5 mm/s. The preparation of the numerical sample consists of two main steps. Firstly, a dense particle assembly using spheres with an average diameter of 7 mm is generated. Secondly, the spheres are randomly replaced by sand and rubber particles based on the specified RC. A typical configuration is shown in Figure 3.

![Figure 3: Typical configuration for preparation and loading of the sample, here for a rubber content of RC=40%.](image)
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2.3 Calibration

The inter-particle friction angle, the contact stiffness ratio and the contact Young’s moduli are microscopic parameters. Hence, they need to be calibrated. The calibration was performed by trial and error by selecting different parameter combinations and trying to match the macroscopic response measured during the experiments. The parameters were calibrated based on the pure sand sample (RC=0%) and the pure rubber particle sample (RC=100%). The contact parameter listed in Table 1 were identified as the best set of parameters to reproduce the macroscopic response (see [7] for more details). In order to further validate the parameter combinations, additional simulations were carried out using various mixtures with different rubber contents. The results are summarised in Figure 4. As can be seen, the numerical predictions agree very well with the experimental results.

Table 1: Parameters used in this study.

<table>
<thead>
<tr>
<th>Material</th>
<th>Inter-particle friction angle (deg)</th>
<th>Contact stiffness ratio</th>
<th>Contact young’s modulus (MPa)</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>23</td>
<td>0.15</td>
<td>1000</td>
<td>2600</td>
</tr>
<tr>
<td>Rubber</td>
<td>35</td>
<td>0.48</td>
<td>1.5</td>
<td>1500</td>
</tr>
<tr>
<td>Wall</td>
<td>30</td>
<td>0.2</td>
<td>200000</td>
<td>-</td>
</tr>
</tbody>
</table>

![Figure 4: Comparison of predicted (solid lines) and measured (dashed lines) stress-strain values for various rubber contents.](image)

3 RESULTS AND DISCUSSION

3.1 Variation of coordination number

The concept of ‘grain-contact coordination number’ introduced in [6] is used. This allows to take into account the average number of grain contacts per grain where grain refers to an
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agglomerate of spherical particles (i.e., sand grain corresponds to rigid agglomerate of spheres and rubber grain corresponds to bonded agglomerate if spheres).

Figure 5a shows the variation of grain-contact coordination number with increasing vertical strain. It can be observed that generally the grain-contact coordination number increases with increasing rubber content. This is clear since rubber grains deform easily under compressive force, leading to more contacts between adjacent grains. In other words, consideration of deformability results in realistic responses. The variation of the coordination number for sand-rubber particle mixtures has also been reported by Lopera Perez et al. [9]. Their results indicate that in samples with rubber particles larger than sand particles, the coordination number decreases with increasing rubber content. However, their simulations did not account for deformability of the rubber particles and, hence, some of their findings might be questionable.

Another approach to investigate the effect of deformability on the number of contacts is to find the average number of contacts between two grains. The variation of this parameter with increasing vertical stress is shown in Figure 5b. The average number of contacts between two grains typically increases with increasing vertical stress. This tendency is more pronounced for samples with higher rubber content. The value is around 1.1 for the pure sand mixture (RC=0%), indicating a point to point contact, while it is around 2.6 for the pure rubber mixture (RC=100%), indicating a surface to surface contact. This observation is in agreement with the real nature of the materials. It is also worthwhile to mention that such conclusions have been stated previously for compressible materials based on experimental results (see e.g. [10]).

![Figure 5: Variation of (a) grain-contact coordination number with increasing vertical strain and (b) average number of contacts between two grains with increasing vertical stress.](image-url)
3.2 Variation of force chains

The force chains are represented as lines of all grain interactions. The colour and thickness of each line represents the magnitude of the contact force. As the contact force increases, the line colour changes to red. Note that for clarity the internal bond interactions of the rubber particles are not considered in the force chains. The force chains of samples with rubber contents RC of 0, 20, 40 and 100% at a vertical stress of 250 kPa are shown in Figure 6. The force chain distributions for RC=0% and RC=20% are very similar but with increasing rubber content the number of interactions increase and force chains become less pronounced. In fact, no clear force chains are developing in the sample with RC=100%, indicating that it acts like a solid rather than a granular material.

Figure 6: Force chain distribution for samples with different rubber content at 250 kPa vertical stress.
3.3 Insights from contact network histograms

Figure 8 shows the polar contact network histograms (rose diagrams) for each type of contacts, i.e., sand-sand (S-S), sand-rubber (S-R) and rubber-rubber (R-R), and for all combined contacts at a stress level of 250 kPa. The data consistently shows that there is no apparent bias in the number of contacts oriented in any particular direction. However, it can clearly be seen that the increase in rubber content results in more contacts. For samples with RC=0% and RC=20%, the number of S-S contacts is obviously higher than the number of R-R contacts and, hence, the sand grains control the behaviour of the sample. For RC=40% and RC=60%, the S-R contacts have the highest contribution. For the pure rubber sample (RC=100%), the behaviour is fully controlled by rubber. From these observations it can be concluded that sandy behaviour is observed for samples with less than 20% rubber content, rubbery behaviour is observed for samples with more than 60% rubber content and samples with rubber contents between 20% and 60% exhibit sand-rubber like behaviour.

4 CONCLUSIONS

Deformability and grain shape are two intrinsic characteristics of sand-rubber particle mixtures which need to be taken into account. This paper considers both features in order to carry out realistic simulations of 1D compression tests of mixtures with different rubber content. The main focus of the study is the micromechanical behaviour of such mixtures.

Grain shape and deformability lead to interlocking of grains. The higher the interlocking, the higher the residual strain. The numerical results are able to predict this phenomenon and the predictions agree very well with experimental observations. Another reason for such good agreement is use of a 3D model. The number of grain-wall contacts in 2D simulations is significantly lower than a 3D which results in lower wall friction and, hence, in lower residual strain (cf. [3]).

The concept of coordination number is used on grain-contact level. The number of grain-contacts generally increase with increasing load. However, the range of this values for pure rubber is about 60% larger than pure sand. Inspired by this observation, the value of average number of contacts between two specific grains is investigated. This shows the change in shape of the contact area from a point to point for sand-sand contacts to surface to surface for rubber-rubber particle contacts. Such observations are consistent with the deformable nature of the rubber particles.

The variations of the force chains show larger number of contacts with lower force magnitude for samples with higher rubber content at the same stress level. More insights are obtained from contact network histograms. The ratio of the number of contacts for each type of contact (sand-sand, sand-rubber and rubber-rubber) to the total number of contacts is a qualitative criteria to see which material controls the behaviour of the mixture. Results indicate that sand is generally controlling the behaviour of mixtures for samples with rubber content lower than 20% while a sample with rubber content of larger than 60% is controlled by rubber.
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<table>
<thead>
<tr>
<th>Sand-sand contact</th>
<th>Sand-rubber contact</th>
<th>Rubber-rubber contact</th>
<th>All contacts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{RC}=0%$</td>
<td>$\text{RC}=20%$</td>
<td>$\text{RC}=40%$</td>
<td>$\text{RC}=100%$</td>
</tr>
</tbody>
</table>

**Figure 7:** Contact network histograms at 250 kPa vertical stress.
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REFERENCES


